The yield stress of opposed anvils

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The Lur'e solution for an elastic cone has been integrated to give a more reasonable solution of the state of stress of a Bridgman anvil. The maximum shear stress at the center axis is plotted as a function of the tapered angle. It shows that with a smaller tapered angle the anvil gives higher pressure before it plastically yields. It is estimated the maximum pressure can be achieved in a Drickamer-type apparatus with pistons made of maraging steel is around 85 kbar and that of cemented tungsten carbide is around 300 kbar. Based on published claims of achieved pressures, the maximum pressure capability of sintered-diamond compact is greater than 1.2 Mbar and that of a single-crystal diamond could possible be as high as 3.2 Mbar.

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I. INTRODUCTION

The yield strength of a Bridgman anvil is an important parameter in estimating the maximum pressure that can be achieved in a Bridgman opposed anvil apparatus or a standard Drickamer-type apparatus. Unfortunately, the materials for making these pistons, such as grade 999 Carboloy$^{\mathrm{T}}$ cemented tungsten carbide, sintered diamond, and single-crystal diamond all fracture in a brittle fashion with negligible plastic deformation. Therefore, the yield strengths of these materials at room temperature are usually unknown. However, the tip of the Bridgman anvil, while under compression with proper support, provides a sufficiently large hydrostatic stress component such that the plastic deformation of these materials is possible. Recently, Ruoff and Wanagel[1] by approximate analysis of the state of stress in supported opposed anvils and by the measurement of the pressure at which the anvil tips exhibit a permanent deviation from planarity were able to estimate the yield stress of the cemented tungsten carbide. It is shown in the present paper that the Lur'e solution[2] they had used can be integrated to give a more reasonable solution of this problem; the maximum shear stress at the center axis can be obtained as a function of the tapered angle of the anvil. In Sec. III the maximum pressure achievable with pistons made of different materials is discussed.

II. THEORETICAL

The standard Drickamer-type apparatus consists of two opposed tungsten carbide pistons placed in a cylinder with the space between them filled with pyrophyllite. The Bridgman opposed anvil apparatus is more or less the same; the difference is that there is no support over the conical flanks of the pistons. The specimen is encased in a circular pyrophyllite ring along with two disks and is placed in between the two anvils. It has been shown by Forsgren and Drickamer[3] that the pressure on the anvil tip is more or less uniform. However, it should be noted that it is only true when the sample thickness (i.e., the gap between the opposed anvils) is very small. It was shown by Myers *et al.*[4] that the pressure distribution over the flat surface of the anvil depends strongly upon the diameter-to-thickness ratio of the sample. Recently, Bundy[5] also showed that the slope of the pressure on the flat surface versus the applied load increases with decreasing gap thickness. Therefore, it can be concluded that in a Drickamer apparatus with a very thin sample, the pressure on the flat is approximately uniform to the edge of the circular flat but then decreases rapidly over the tapered surface. However, for a thick sample, the pressure on the flat has a Gaussian-type distribution.[6] The latter is also true for opposed anvil apparatus. In the present paper, we will consider a situation where the flat surface of radius a of the anvil is subject to a uniform pressure as shown in Fig. 1(a). This approximates the case of a very thin sample. We shall now proceed to find the state of stress inside the anvil.

Lur'e[2] has shown the elastic solution for the case in which a point load is applied at the vertex of a cone and acts inward along the axis of the cone. His solution in spherical coordinates is as follows:

$$\begin{aligned}
\sigma_R &= \frac{C}{R^2}(1+\cos\gamma - A\cos\theta),\\
\sigma_\theta &= \frac{C}{R^2}\left(\frac{\cos\theta(\cos\theta-\cos\gamma)}{1+\cos\theta}\right),\\
\sigma_\phi &= \frac{C}{R^2}\left(\frac{\cos\theta-\cos\gamma}{1+\cos\theta} - 1 + \cos\theta\right),\\
\sigma_{R\theta} &= \frac{C}{R^2}\left(\frac{\sin\theta(\cos\theta-\cos\gamma)}{1+\cos\theta}\right),
\end{aligned} \tag{1}$$

where

$$C = \frac{Q(m-2)}{8\pi(m-1)},$$

$$Q = \frac{4F(m-1)}{m(1-\cos^3\gamma)-(m-2)\cos\gamma(1-\cos\gamma)},$$

$$A = \frac{4m-2}{m-2},$$

and $m = 1/\nu$, ν is Poisson's ratio, γ is the half-apex-angle of the cone, F is the force acting along the cone axis, and R and θ are shown in Fig. 1(b). Now, in order to obtain a solution for the case of Fig. 1(a), we integrate the Lur'e solution[2] with the vertex of the cone tracing out a circular area with the diameter equal to that in Fig. 1(a). This is the same as Timoshenko and Goodier[7] did for the solution of the Boussinesq problem. For convenience in later analysis, we shall transform the stress tensor components to a cylindrical

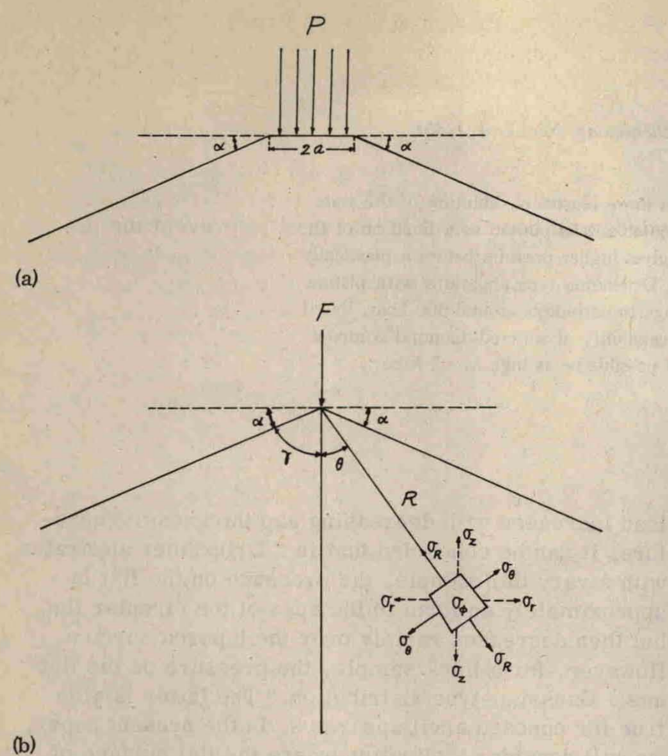

FIG. 1. (a) Side view of anvil with uniform loading. (b) Cone with a point load at the vertex.

coordinates by using the following equations:

$$\begin{aligned}
\sigma_z &= \sigma_R \cos^2\theta + \sigma_\theta \sin^2\theta - 2\sigma_{R\theta}\sin\theta\cos\theta,\\
\sigma_r &= \sigma_R \sin^2\theta + \sigma_\theta \cos^2\theta + 2\sigma_{R\theta}\sin\theta\cos\theta,\\
\sigma_\phi &= \sigma_\phi,\\
\sigma_{rz} &= (\sigma_R - \sigma_\theta)\cos\theta\sin\theta + \sigma_{R\theta}(\cos^2\theta - \sin^2\theta).
\end{aligned} \tag{2}$$

After integration, we obtain the stress distribution along the z axis as follows:

$$\begin{aligned}
\sigma_z &= 2\pi C[-\tfrac{1}{3}(A-1)(1-\cos^3\beta) + \cos\gamma(1-\cos\beta)],\\
\sigma_r = \sigma_\phi &= \pi C[\tfrac{1}{3}(A-1)(1-\cos^3\beta)\\
&\quad + (2 - A - \cos\gamma)(1-\cos\beta)],
\end{aligned} \tag{3}$$

where

$$\cos\beta = [z/(a^2+z^2)^{1/2}].$$

When $\gamma = 90°$, this solution becomes exact and reduces to that of Timoshenko and Goodier[7] on the Boussinesq problem, i.e., a uniform pressure acting normally on a circular area of radius a on the planar boundary of a semi-infinite medium. For γ slightly less than 90°, or small tapered angle α ($\alpha = 90° - \gamma$), this solution should be very close to the exact solution.

It is easily shown that the shear stress $\frac{1}{2}(\sigma_\phi - \sigma_z)$ becomes a maximum along the z axis at a depth

$$z = a[k/(1-k)]^{1/2}, \tag{4}$$

where

$$k = \frac{(A + 3\cos\gamma - 2)}{3(A-1)},$$

and the shear stress there is

$$\tfrac{1}{2}(\sigma_\phi - \sigma_z)_{\max} = \tfrac{1}{2}\pi C[(A-1)(1-\cos^3\beta) + (2 - A - 3\cos\gamma)(1-\cos\beta)]. \tag{5}$$

If we use the maximum yield stress criterion of Tresca, then yielding occurs when $(\sigma_\phi - \sigma_z)_{\max} = \sigma_0$, where σ_0 is the yield strength of the material. With this analytic solution, it enables us to plot the ratio $(\sigma_\phi - \sigma_z)_{\max}/\sigma_{z=0}$ as a function of the tapered angle. It is shown in Fig. 2 for the case of cemented tungsten carbide with $\nu = 0.185$. From the curve, we see that with a smaller tapered angle, one can achieve higher pressure before yielding. However, with a very small angle (a flat being the limit) and a very small circular flat, one should note that the increase of contact area due to elastic deformation of the tip would limit the maximum pressure geometrically.

From the solution we obtained, we see that σ_z, σ_r, and σ_ϕ are all negative at the tip of the anvil. σ_r and σ_ϕ are only slightly less in magnitude than σ_z; because of this, there is a large hydrostatic component at the tip which prevents the material from brittle fracture and also makes the plastic deformation possible.

III. DISCUSSION

Now, we shall assume a standard configuration of a Drickamer-type apparatus. i.e., tapered angle $\alpha = 18°$, and proceed to calculate the maximum pressure before yielding for anvils of different materials. We shall estimate the maximum pressure obtainable with opposed anvils made of maraging steel, cemented tungsten carbide, single-crystal diamond, and sintered diamond.

For maraging steel, we have $\nu = 0.30$. When substituted in Eqs. (3)–(5), we obtain $\sigma_0 = 0.666\sigma_{z=0}$ and yielding starts at $z = 0.702a$. With the known yield stress for maraging steel equal to 20 kbar, we would expect the pressure at the onset of the plastic deformation at around 30 kbar.

For cemented tungsten carbide, we use the value $\nu = 0.185$ obtained from elastic constant measurement

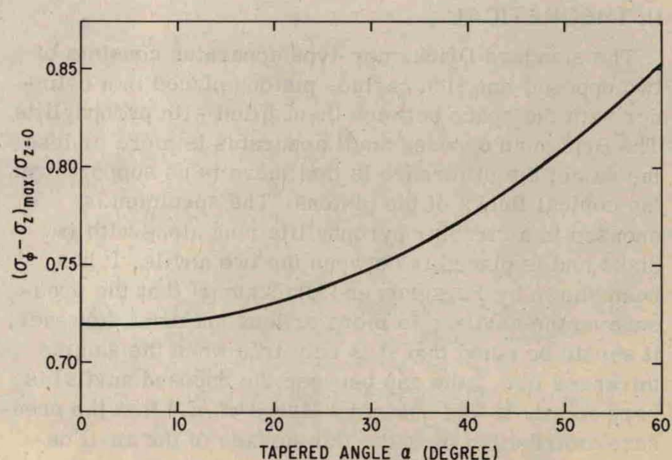

FIG. 2. The ratio of $(\sigma_\phi - \sigma_z)_{\max}$ to σ_z at $z = 0$ as a function of tapered angle α.

by Day and Ruoff.[8] We find that $\sigma_0 = 0.729\sigma_{z=0}$ and yielding begins at the point $z = 0.699a$. If the onset of plastic deformation really starts at a pressure around 110 kbar as mentioned in Ref. 1, the yield strength of cemented tungsten carbide is around 80 kbar. The above result for maraging steel and cemented tungsten carbide is only slightly different from that of Ruoff and Wanagel,[1] although the latter used a relatively crude analysis.

If we had used the Poisson ratio for cemented tungsten carbide from the direct measurement[9] rather than that obtained from the ultrasonic work,[8] namely, $\nu = 0.19-0.23$, we would have $\sigma_0 = 0.726\sigma_{z=0}-0.704\sigma_{z=0}$. Assuming the same value of pressure for the onset of the plastic deformation for the anvils, we obtained the yield strength of cemented tungsten carbide around 80—77 kbar. The direct measured yield strength based on 0.002% offset is approximately 350 000 psi (~24 kbar).[9] This should provide the lower bound of the generally defined yield strength based on 0.2% offset.

For single-crystal diamond, we have used $\nu = 0.103$ for Poisson's ratio, which is obtained from the data of the adiabatic elastic constants measured by McSkimin *et al.*[10] and converted to isothermal ones. We find that $\sigma_0 = 0.781\sigma_{z=0}$ and yielding begins at $z = 0.697a$.

Now, it would be interesting to estimate the maximum pressure one can achieve with the Drickamer-type apparatus by using pistons of these different materials. For pistons made of maraging steels, Ruoff and Wanagel[11] claimed that a maximum pressure of around 85 kbar was obtained, i.e, approximately four times the yield strength of the marging steel; this result is quite interesting, because with strong enough support along the conical flanks of the pistons, one can imagine that toward the center of the highly pressurized zone the state of stress can be approximated by a hollow sphere pressurized inside. Then, the well-known result from elastic and plastic theory that $P = 2\sigma_0 \ln K$ (assuming no work hardening), where K being the radius ratio, would tell us that with $K = 7$ it would give us a maximum pressure roughly four times the yield strength. This is actually the case too with cemented tungsten carbide pistons of $K = 10$, which is a usual design figure for a standard Drickamer-type apparatus, that after heavy deformation one usually has a plastic zone around $K = 7$ or less. If the value of yield strength is correct, namely, 80 kbar, then using cemented tungsten carbide one would obtain a maximum pressure around 300 kbar with $K = 10$. However, it seems that the determination of the onset of plastic yielding by measuring the permanent deformation at the tip of the piston is not a very sensitive method. The estimated yield strength could possibly be lower. And also, due to the fact that stress/strain curves for steel and cemented tungsten carbide are not exactly the same, the analysis made here about the maximum pressure is a rough estimate.

It is understood that in order to effectively use the load toward the center area without too much plastic deformation along the conical flanks of the anvils, one usually uses the optimum design figure $K = 10$. Hence, the maximum achievable pressure estimated here will be based on the value of $K = 10$, which would generally allow a fully plastic zone of $K = 7$ roughly. One certainly can use a value of K much larger than 10. Then he has to provide extremely strong support along the conical flanks of the anvils in order to have a large enough hydrostatic component in that area to prevent the piston from failing. In the latter case, however, a larger fraction of the total load will be taken by the conical flanks of the anvils.

As for the case of a single-crystal diamond, there is no available data on the yield strength. However, it is known for some cases in an indentation test, it could stand pressure as high as 300 kbar. Although the situation is not completely identical to the anvils we consider here, one can roughly estimate a maximum achievable pressure of at least 1.2 Mbar. The recent Mao and Bell[12] experiment with single-crystal-diamond anvils indicated that a pressure of 1.018 Mbar was obtained without any deformation of the diamond. If the claimed pressure is accurate, that would mean a yield strength of 800 kbar which is approximately one-seventh of the shear modulus of diamond. Then the maximum achievable pressure with a single-crystal diamond could be as high as 3.2 Mbar.

Bundy[5] in a recent experiment with a sintered-diamond tip on a cemented tungsten carbide piston has achieved a pressure of approximately 400 kbar without any measurable plastic deformation at the tip. No Poisson rate is available for the sintered-diamond compact; but if the same equation for a single-crystal diamond is used, one can estimate a maximum achievable pressure of at least 1.2 Mbar. Since the sintered-diamond anvils have not been tested experimentally to determine onset of plastic deformation but do show indentation hardness values almost equivalent to those obtained for single-crystal diamond, the ultimate pressure capability may be as high as for a single-crystal diamond.

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[1]A.L. Ruoff and J. Wanagel, J. Appl. Phys. 46, 4647 (1975).
[2]A.I. Lur'e, *Three-Dimensional Problems of the Theory of Elasticity* (Wiley, New York, 1964), p. 137.
[3]K.F. Forsgren and H.G. Drickamer, Rev. Sci. Instrum. 36,

1709 (1966).
[4]M. Myers, F. Dachille, and R. Roy, *High Pressure Measurement*, edited by A. A. Giardini and E. C. Lloyd (Butterworth, Washington, D.C., 1963), p. 17.
[5]F. P. Bundy, Rev. Sci. Instrum. 46, 1318 (1975)
[6]D. Papantonis, M.S. thesis (University of Rochester, 1975) (unpublished).
[7]S. Timoshenko and J. N. Goodier, *Theory of Elasticity* (McGraw-Hill, New York, 1970), p. 403.
[8]J. P. Day and A. L. Ruoff, J. Appl. Phys. 44, 2447 (1973).
[9]R. C. Lueth (private communication).
[10]H. J. McSkimin, P. Andreatch, Jr., and P. Glynn, J. Appl. Phys. 43, 985 (1972). Poisson's ratio in this case actually varies with crystallographic orientation. We have obtained the value from $\nu_{21} = C_{12}/(C_{11} + C_{12})$.
[11]A. L. Ruoff and J. Wanagel, Rev. Sci. Instrum. 46, 1294 (1975).
[12]H. K. Mao and P. M. Bell, Science 191, 851 (1976).